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MHD Stagnation Point Flow and Heat Transfer of Williamson Fluid over Exponential Stretching Sheet Embedded in a Thermally Stratified Medium

Ch.Vittal[1*], M. Chenna Krishna Reddy[2] and T.Vijayalaxmi[2]

[1,2] Department of Mathematics, Osmania University, Hyderabad, 500007, Telangana, India.*
[2] MVS Govt Arts & Science UG, PG College Christianpally, Mahaboobnagar, 509001, Telangana, India.

E-mail: vittalch1006@gmail.com, ckreddym@yahoo.com, vijaya9966998024@rediffmail.com

Abstract

Magnetohydrodynamics (MHD) stagnation point flow and heat transfer of a Williamson fluid in the direction of an exponentially stretching sheet embedded in a thermally stratified medium subject to suction present in this examination. Suitable transformations are used to convert the partial differential equations corresponding to the momentum and energy equations into highly nonlinear ordinary differential equations. The resulting equations are successfully solved by using an implicit finite difference scheme known as Keller-Box method. The results revealed that the velocity enhances with Williamson parameter and temperature reduces with stagnation parameter. The heat transfer rate at the surface increases in the presence of thermal stratification. Fluid velocity decreases with increment in magnetic parameter.

Keywords: “Williamson fluid”, “Stagnation point”, “Exponentially stretching sheet”, “MHD”, “Suction”, “Boundary layer flow”, “Thermally stratified medium”.

I. INTRODUCTION:

Williamson fluid is a pseudoplastic fluid and belongs to Non-Newtonian fluid. Study of the boundary layer flow of pseudoplastic fluid is becoming important due to its great interest of wide range of application in industry. To explain the behavior of pseudoplastic fluid models like power law model, Carreaus model, Cross model and Ellis model, etc. are proposed. In 1929, Williamson consider the flow of pseudoplastic materials and proposed a model with equations to describe the flow of pseudoplastic fluids and it has been verified experimentally. Heat and mass transfer effects with the peristaltic flow of Williamson fluid in a vertical annulus is studied by Nadeem et al. [1]. Vajravelu et al. [2] discussed the peristaltic transport of a Williamson fluid with permeable walls in asymmetric channel. Investigations on the Williamson fluid model under different flow patterns are discussed. Dapra et al [3] reported the perturbation solution for a Williamson fluid which injected into a rock fracture. Alam Khan et al [4] studied the series of solutions using of homotopy analysis method (HAM) in four flow problems of a Williamson fluid. Nadeem group [5] performs the modelling of a two-dimensional flow analysis for Williamson fluid over a linear and exponentially stretching surface. Hayat team[6] studied the series solution for the time independent MHD flow of Williamson fluid past a porous plate.

Stratification effect has great importance in engineering processes and for applications like metallurgy, hydro magnetic methods, drying processes, solar collectors, polymer production. It is an important aspects to consider in the study of heat transfer. Stratification of fluids occurs due to the temperature variations or concentration differences, or the presence of different fluids of different densities. Rosmila et al [7] studied the magnetic effect on the natural convective flow of a nanofluid, over a linearly porous stretching sheet in the presence of thermal stratification. Yang group [8] studied the laminar free convection from a non-isothermal plate immersed in a temperature stratified medium. Jaluria et al [9] reported the stability and transition of buoyancy induced flows in a stratified medium. Chen [10] reported the flow due to a heated surface immersed in a stable stratified viscous fluid. Nevertheless, convective flow in a stratified medium has not received much attention.

In addition, the stagnation point flow is an interesting area of research with applications in industrial and scientific area. Mahapatra [11] reported the boundary layer is formed when the stretching velocity is less than a free stream velocity and an inverted boundary layer is formed when the stretching velocity exceeds the free stream velocity. Wang his team [12] reported the stagnation point flow when the line of stagnation is perpendicular to the stretching surface. Lok et al. [13] numerically studied non-orthogonal stagnation point flow towards a stretching sheet using Keller-box method, results in obliqueness of a free stream line causes the shifting of the stagnation point towards the incoming flow. The study of a stagnation point flow towards a solid surface in moving fluid traced back to Hiemenz [14].

The study of magneto-hydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal working processes. The process of fusing of metals in an electrical furnace by applying a magnetic field and the process of cooling of the first wall inside a nuclear reactor containment vessel where the hot plasma is isolated from the wall by applying a magnetic field are some examples of such fields (Ibrahim[15]. In controlling momentum and heat transfers in the boundary layer flow of different fluids over a stretching sheet, applied magnetic field may play an important role (Turkyilmazoglu, [16]. Kumaran et al. [17] reported that magnetic field makes the streamlines steeper which results the boundary layer thinner.

In the present paper, the MHD stagnation point flow effect and heat transfer of Williamson fluid over exponential stretching sheet which are embedded in a thermally stratified medium is studied using Keller-Box method and the results are discussed.

II. FLUID MODEL:

For the Williamson fluid model, the Cauchy stress tensor *S* is defined as

$$S = -pI + \tau \qquad (1)$$

$$\tau = \left(\mu_\infty + \frac{\mu_0 - \mu_\infty}{1 - \Gamma \dot{\gamma}} \right) A_1 \qquad (2)$$

where τ is the extra stress tensor, μ_0 is the limiting viscosity at zero shear rate, μ_∞ is the limiting viscosity at the infinite shear rate, $\Gamma > 0$ is a time constant, $A1$ is the first Rivlin-

Erickson tensor, and $\dot{\gamma}$ is defined as

$$\dot{\gamma} = \sqrt{\frac{1}{2}\pi}, \quad \pi = trace(A_1^2). \qquad (3)$$

Here, we consider the case in which $\mu_\infty = 0, \quad \Gamma \dot{\gamma} > 1.$

Finally, we get, $\tau = \frac{\mu_0}{1 - \Gamma \dot{\gamma}} A_1 \qquad (4)$

or $\tau = \mu_0 (1 + \Gamma \dot{\gamma}) A_1. \qquad (5)$

III. FORMULATION OF THE PROBLEM:

Consider the flow of an incompressible viscous electrically conducting Williamson fluid past a flat heated sheet coinciding with the plane y = 0. The flow is confined to y > 0. Two equal and opposite forces are applied along the x-axis, so that the wall is stretched keeping the origin fixed (Fig. 1) with the velocity $U = U_W = be^{\frac{x}{L}}$, also the ambient fluid velocity is $U_\infty = ae^{\frac{x}{L}}, \; a > 0, \; b > 0.$ A variable magnetic field $B = B_0 e^{\frac{x}{2L}}$ is applied normal to the sheet, B_0 being a constant (Ishak, [18]. The sheet is of temperature $T_W(x)$ and is embedded in a thermally stratified medium of variable ambient temperature $T_\infty(x)$ where $T_W(x) > T_\infty(x)$. It is assumed that $T_W(x) = T_0 + ce^{\frac{x}{2L}}, \;\; T_\infty(x) = T_0 + de^{\frac{x}{2L}}, where\ T_0$ is the reference temperature, $c > 0, d \geq 0$ are constants.

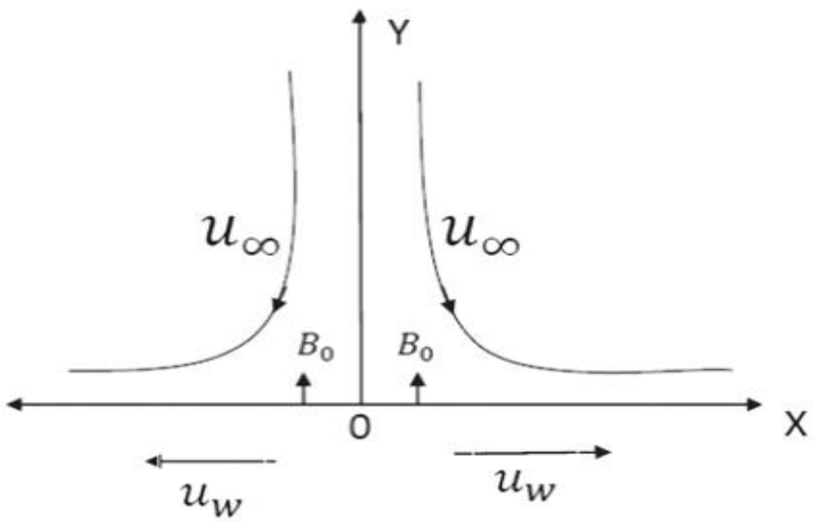

Figure 1. Flow organization with coordinate system.

The continuity, momentum, and energy equations governing such type of flow are written as (Nadeem, [19]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (6)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + U_\infty\frac{\partial U_\infty}{\partial x} + \sqrt{2}\upsilon\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}\left(u - U_\infty\right) \qquad (7)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \qquad (8)$$

Where $u\ and\ v$ are the components of velocity in the x and y directions respectively, $\upsilon = \frac{\mu}{\rho}$ is the kinematic viscosity, c_p is the specific heat at constant pressure and

$\alpha = \dfrac{k}{(\rho C)_f}$ is the thermal diffusivity, ρ is the fluid density and μ is the coefficient of fluid viscosity. Nielsen and Balling studied the thoroughly about horizontally stratified medium (Nielsen[20].

Using the Rosseland [21] approximation as in Cortell [22], the radiative heat flux is simplified as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \qquad (7)$$

We assume that the temperature differences within the flow region, namely, the term T^4 can be expressed as a linear function of temperature. The best linear approximation of T^4 is obtained by expanding it in a Taylor series about $T\infty$ and neglecting higher order terms. That is

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \qquad (8)$$

using eqs (8) into eqs (7), the modified eqs of (6) is

$$q_r = \frac{-4\sigma^*}{3k^*}\frac{\partial}{\partial y}\left(4T_\infty{}^3 - 3T_\infty{}^4\right) = \frac{-16\sigma^* T_\infty{}^3}{3k^*}\frac{\partial T}{\partial y} \;\; and \;\; \frac{\partial q_r}{\partial y} = \frac{-16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} \qquad (9)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(1 + \frac{4}{3}R\right)\frac{\partial^2 T}{\partial y^2} \qquad (10)$$

The proper boundary conditions for the problem are given by

$$\begin{aligned} &u = U_W, \quad V = -V(x), \quad T = T_W(x) \quad && at \quad y = 0, \\ &u \to U_\infty, \quad T \to T_\infty && at \quad y \to \infty \qquad (11) \end{aligned}$$

$V(x) > 0$ is velocity of suction and $V(x) < 0$ is velocity blowing, $V(x) = V_0 e^{\frac{x}{L}}$ is a special type of velocity at the wall is considered. V_0 is the initial strength of suction.

By introducing the suitable similarity transformations (Magyari,[23]

$$\eta = \sqrt{\frac{b}{2\upsilon L}}\, e^{\frac{x}{2L}} y, \;\; u = b e^{\frac{x}{L}} f'(\eta), \;\; v = -\sqrt{\frac{\upsilon b}{2L}} e^{\frac{x}{2L}}\left(f(\eta) + \eta f'(\eta)\right), \;\; G(\eta) = \frac{T - T_\infty}{T_W - T_0} \qquad (12)$$

nd upon substitution of eqs (12) in eqs (5) and (10) the governing equations transforms to

$$f''' + f\, f'' + 2\left(\varepsilon^2 - (f')^2\right) + \lambda\, f''\, f''' - M(f' - \varepsilon) = 0 \qquad (13)$$

$$\left(1 + \frac{4}{3}R\right)G'' + \Pr\left(f\, G' - f' G\right) - \Pr St f' = 0 \qquad (14)$$

The transformed boundary conditions are

$$f' = 1,\ \ f = S,\ \ G = 1 - St \ \ at\ \ \eta = 0,$$
$$f' \to \varepsilon,\ G \to 0,\ \ as\ \eta \to \infty \qquad (15)$$

where the prime denotes differentiation with respect to η, $M = \frac{2\sigma B_0^2 L}{\rho b}$ is the magnetic parameter, $\varepsilon = \frac{a}{b}$ is the stagnation (velocity ratio) parameter, $\lambda = \Gamma\sqrt{\frac{b^3 e^{\frac{3x}{L}}}{\upsilon L}}$ is the Willimson parameter, $S = \frac{V_0}{\sqrt{\frac{bv}{2L}}} > 0\,(or < 0)$ is the suction (or blowing) parameter, $St = \frac{d}{c}$ is the stratification parameter, $St > 0$ implies a stably stratified environment, while $St = 0$ corresponds to an un stratified environment, $R = \frac{4\sigma^* T_\infty^3}{k^* \alpha(\rho c)_p}$ is the thermal radiation and $\Pr = \frac{\upsilon}{\alpha}$ is the Prandtl number.

The Skin friction coefficient and Nusselt number are given by

$$C_f = \frac{\tau_w}{\rho U_w^2},\ \ Nu_x = \frac{L q_w}{k(T_w - T_\infty)}\ \ where\ \ \tau_w = \mu_0\left(\frac{\partial u}{\partial y} + \frac{\Gamma}{\sqrt{2}}\left(\frac{\partial u}{\partial y}\right)^2\right)_{y=0},\ \ q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad (16)$$

q_w *is the heat flux at the surface,* k *is the thermal conductivity of the fluid.*

By substituting eqs (12) into eqs (16) we will get

$$\sqrt{2\,\mathrm{Re}}\,C_f = (f''(0) + \frac{\lambda}{2} f''^2(0)),\ \ Nu_x\sqrt{\frac{2}{\mathrm{Re}}}(1 - St) = -G'(0) \qquad (17)$$

Where, $\mathrm{Re} = \frac{U_w L}{\upsilon}$ local Reynolds number.

IV. NUMERICAL METHOD:

The ordinary differential eqs (13), (14) with the boundary conditions of eqs (15) are solved numerically by using of Keller-Box method, as revealed by (Keller, [24], Cebeci [25] the following few steps are involved to achieve Numerical solutions:

- Reduce the above mentioned higher order ordinary differential equations into a system of first order ordinary differential equations;
- Write the finite differences for the first order equations.
- Linearize the algebraic equations by Newton's method, and write them in matrix–vector form; and
- Solve the linear system by the block tri-diagonal elimination technique.

To get the accuracy of this method the appropriate initial guesses have been chosen.

The following initial guesses are chosen.

$$f_0(\eta) = (1+S-\varepsilon) + \varepsilon x + (\varepsilon - 1)e^{-x}, \quad G_0(\eta) = (1-St)e^{-x}.$$

V.RESULTS AND DISCUSSION:

The transformed momentum and energy equations subjected to the boundary conditions were numerically solved by using the Keller Box method. Fig. 2 to 13 are plotted for the velocity and temperature profiles for different values of governing parameters. In order to find the accuracy of our work, a comparison has been made with the previous results and we obtained excellent agreements which are displayed in Table 1. i.e. the results for heat transfer coefficient is compared with results of Bidin [26] when thermal stratification and suction are absent. Moreover, the values of the skin friction coefficient and the local Nusselt number of different parameters are given in Tables 2 and 3.

Table 1. Comparison values of $-G'(0)$ for various values of Prandtl number in the absence of remaining parameters.

Pr	Bidin[26]	S. Mukhopadhyay[27]	Present study
1	0.9547	0.9547	0.9548
2	1.4714	1.4714	1.4715
3	1.8961	1.8961	1.8691

The influence of the velocity ratio parameter ε is displayed in Fig. 2. The behavior of ε, which denotes the ratio of free stream velocity to the velocity of the stretching sheet on the velocity field can be observed from Fig. 2. The velocity of the fluid and the boundary layer thickness increases when free stream velocity is less than the velocity of the stretching sheet $(\varepsilon < 1)$ with an increase in ε. However when free stream velocity exceeds the velocity of the stretching sheet $(\varepsilon > 1)$, the velocity of the fluid increases where as the boundary layer thickness decreases with an increase in ε. From Fig. 3 we can observe that as the values of velocity ratio parameter ε increases, the thermal boundary layer thickness decreases. If $\varepsilon = 1$ i.e., when the stretching and free stream velocities are equal, then there is no boundary layer of fluid flow near the sheet. In addition to this the temperature gradient at the surface increase (in absolute value) as ε increases. As a result, local Nusselt number on the surface of a plate increases.

Fig. 4 and 5 shows the variation of Williamson parameter λ on velocity profiles. It can be observed that the velocity decreases with increase in Williamson fluid parameter λ; because after increasing Williamson fluid parameter λ the fluid offers more resistance to flow which decreases velocity. Also from Fig. 5 the momentum boundary layer thickness decreases with the increase in the Williamson parameter.

We can observe the velocity profiles for the variation of magnetic parameter M from Fig. 6. As the values of M increases, the fluid velocity is found to decrease. Actually, rate of transport decreases with the increase in M because the Lorentz force which opposes the motion of fluid increases with the increase in M.

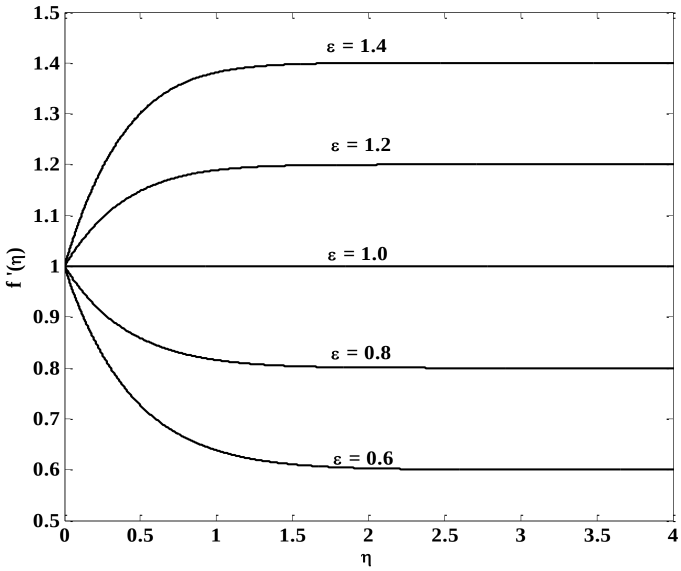

Figure 2. Variation of horizontal velocity f ′(η) with η for several values of stagnation parameter ε.

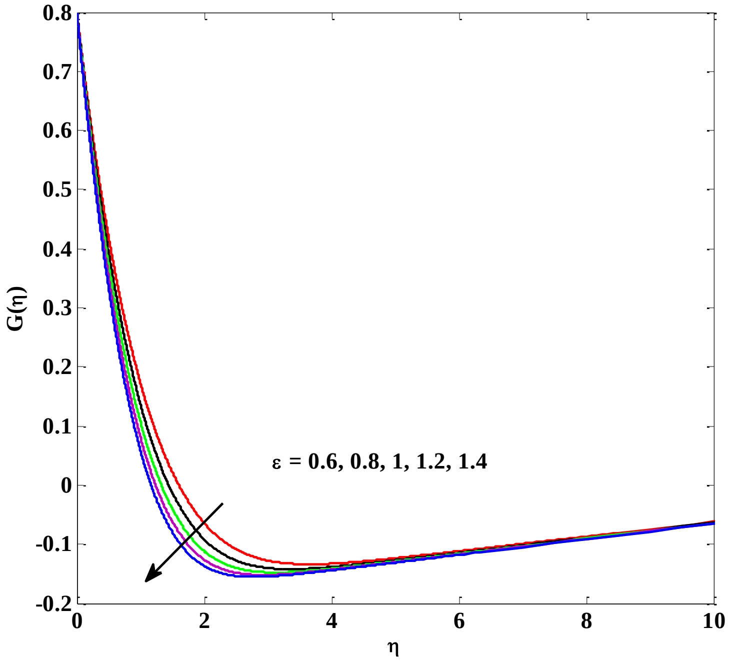

Figure 3. Variation of temperature G(η) with η for several values of stagnation parameter ε.

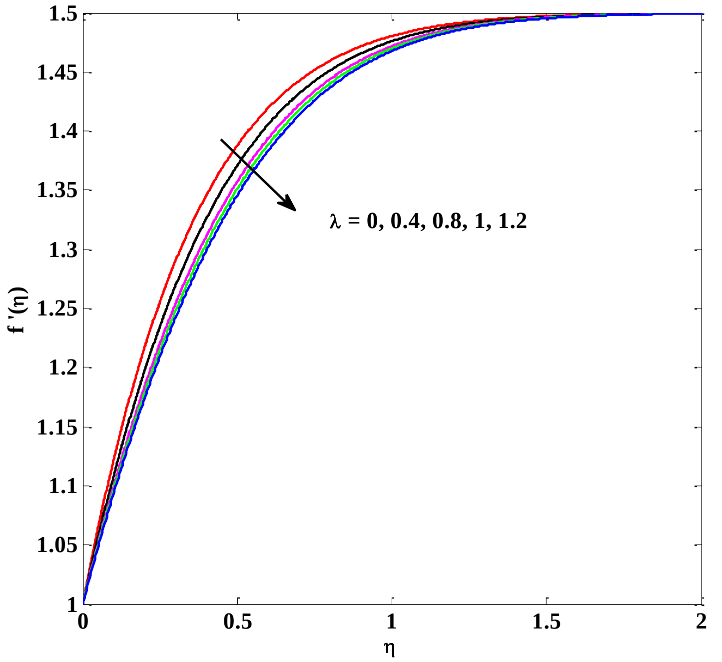

Figure 4. Variation of horizontal velocity f ′(η) with η for several values of Williamson parameter λ.

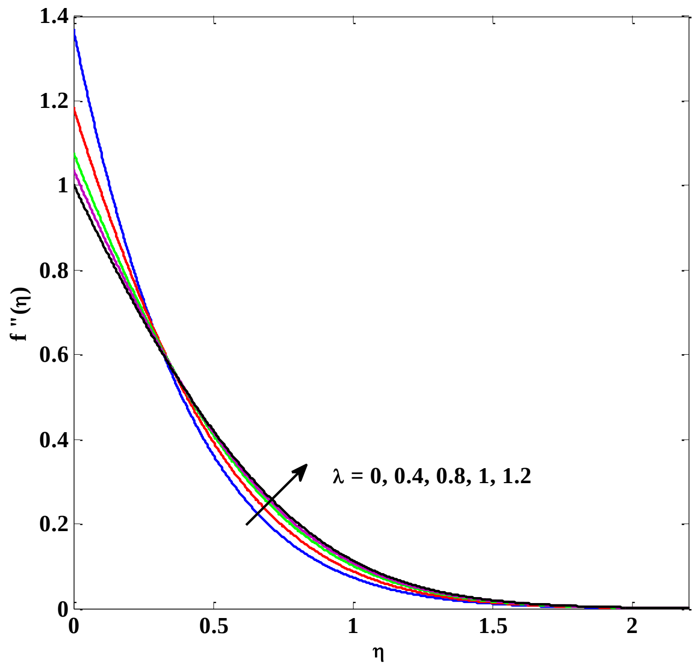

Figure 5. Variation of shear stress f ′′(η) with η for several values of Williamson parameter λ.

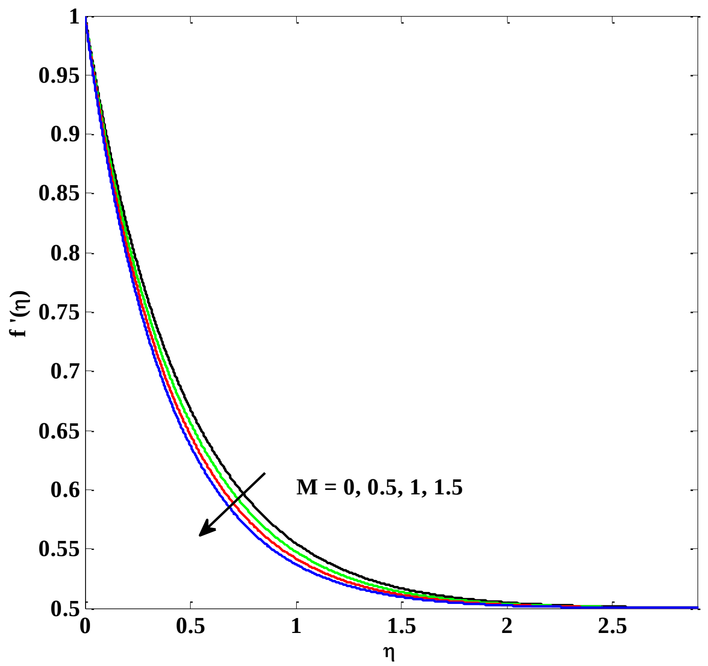

Figure 6. Variation of horizontal velocity f ′(η) with η for several values of magnetic parameter M.

For exponential stretching sheet Fig. 7 and 8 describe the properties of suction parameter S on velocity and shear stress profiles, respectively. We can note that from Fig. 7 as suction parameter increases the velocity decreases a lot. In the case of shear stress it is initially decreasing with suction S, but after a certain distance η from sheet shear stress increases gradually. We can observe this fact From Fig. 8. This is due to consideration of wall suction. We can examine the effect of suction parameter S on temperature and temperature gradient profiles from Fig. 9 and Fig. 10. It is seen that temperature decreases with increasing suction parameter (Fig. 9). Similarly as in the case of shear stress here also, the temperature gradient decreases initially with the suction parameter S, but it increases after a certain distance η from the sheet. This result is shown in Fig. 10. Thus, suction at the surface has a tendency to reduce both the hydrodynamic and thermal boundary layer thicknesses.

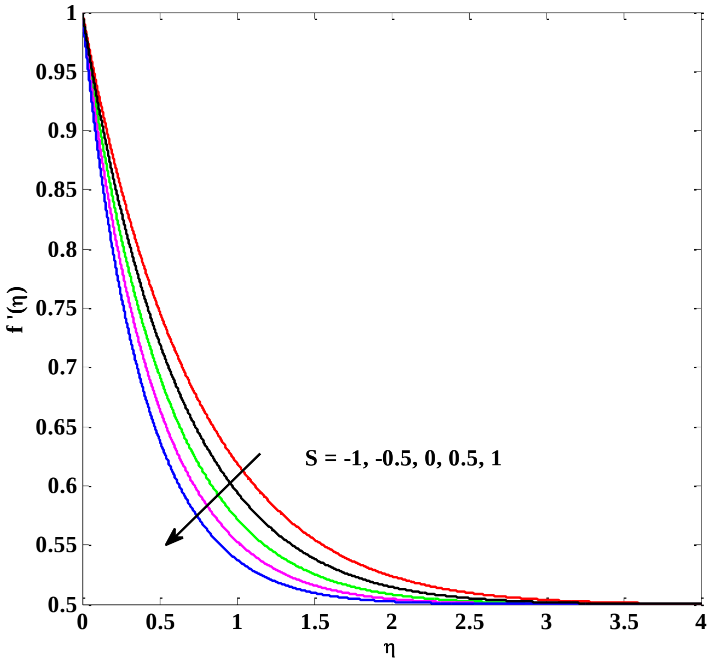

Figure 7. Variation of horizontal velocity f ′(η) with η for several values of suction parameter S.

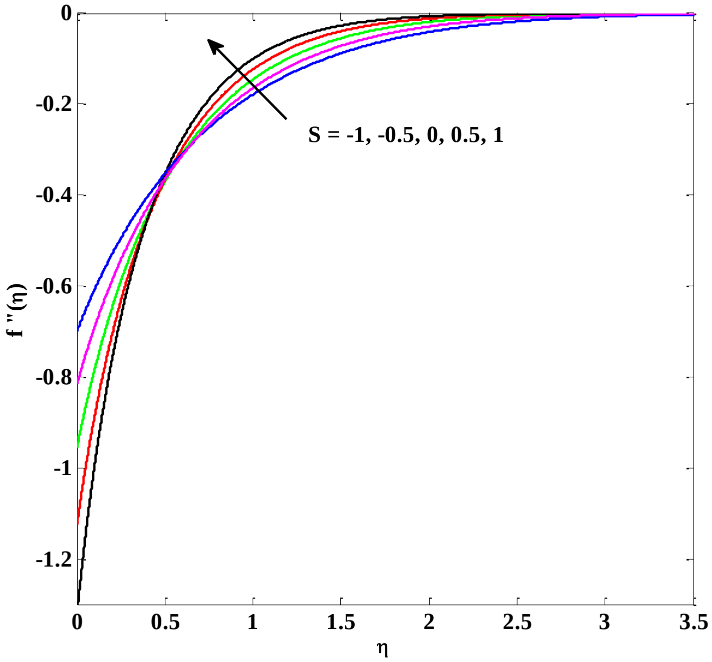

Figure 8. Variation of shear stress f ″(η) with η for several values of suction parameter S.

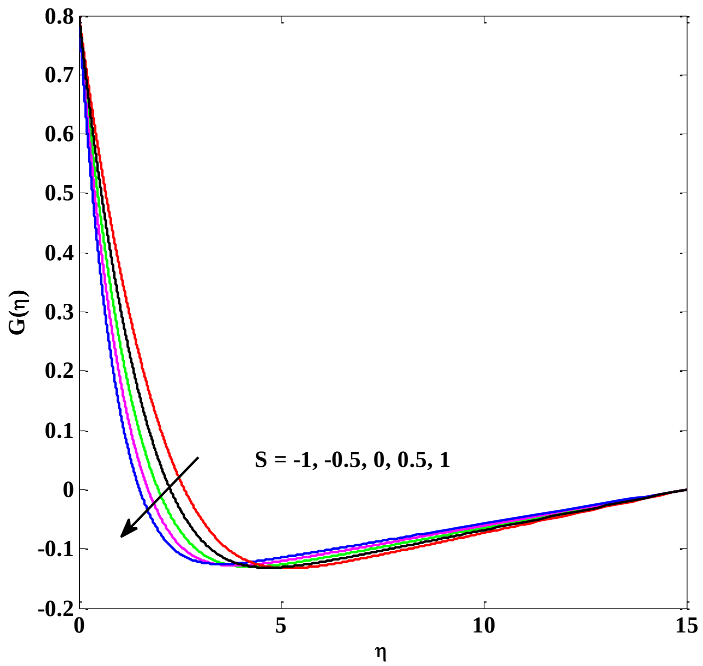

Figure 9. Variation of temperature G(η) with η for several values of suction parameter ε.

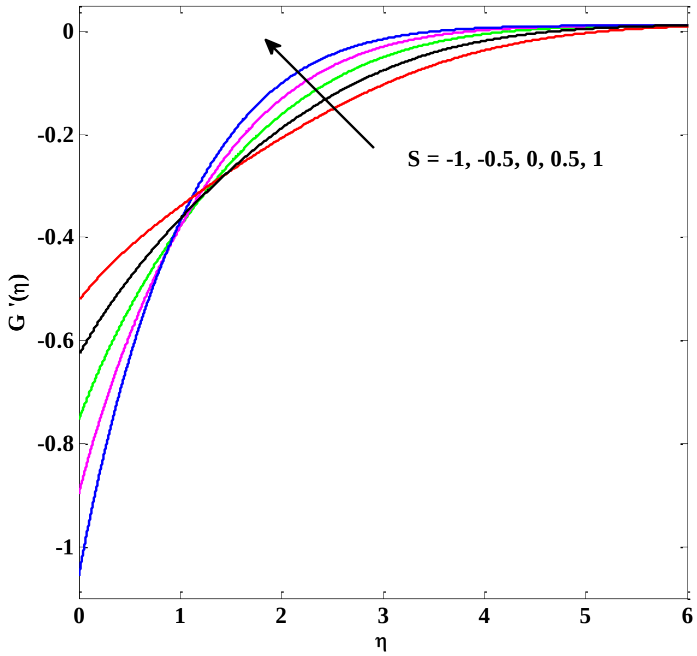

Figure 10. Variation of temperature gradient G ′(η) with η for several values of suction parameter ε.

The effect of thermal stratification parameter (St) on the temperature profile in the presence of suction and stagnation at the boundary is presented in Fig. 11(a). Temperature profile G (η) for different values of the stratification parameter (St) in the absence of suction and absence of stagnation are presented in Fig. 11b. Furthermore the temperature profile for the case of presence of suction without stagnation effect and also for the case of stagnation without suction is presented in Fig. 11c and Fig.11d. It is found that, in all the above cases (Fig. 11a, 11b, 11c and 11d) temperature decreases as the stratification parameter St increases. Since the reason is as St, increases the temperature in free-stream increases or the temperature decreases in the surface level. Thermal boundary layer thickness is therefore also decreased with an increase in St Values. Fig. 12a is the graphical representations of temperature gradient profiles G′ (η) for several values of stratification parameter for the porous sheet (for $S > 0$) and with stagnation. In the case of non-porous ($S = 0$) sheet, without stagnation the effect of Stratification for temperature gradient is reported in Fig. 12b. Furthermore the temperature gradient profile for the case of presence of suction without stagnation effect and also for the case of stagnation without suction is presented in Fig. 12c and Fig.12d. All the temperature profiles decay from the maximum value which is at the wall and tends to zero in the free stream. That is, converge at the outer edge of the boundary layer. The temperature gradient increases considerably with an increase in stratification, St for all cases (Fig 12a, 12b, 12c and 12d).

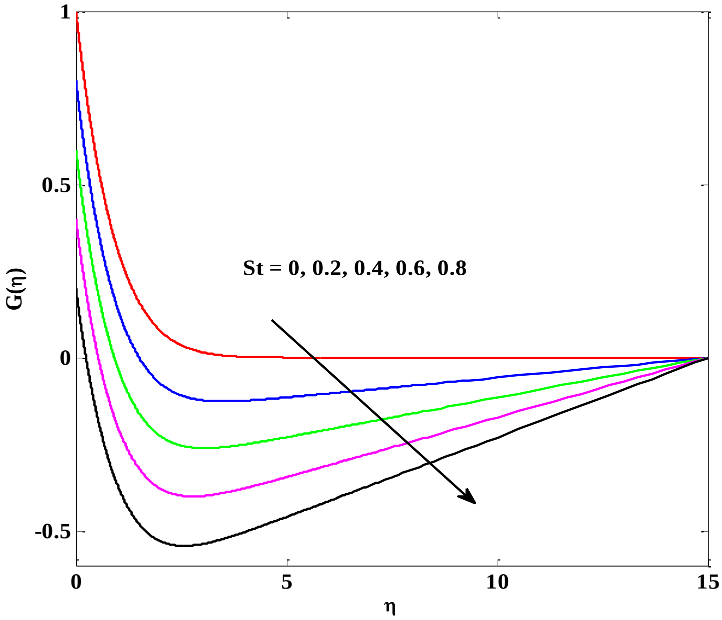

Figure 11(a). Variation of temperature G(η) with η for several values of stratification parameter St in the presence of suction and stagnation.

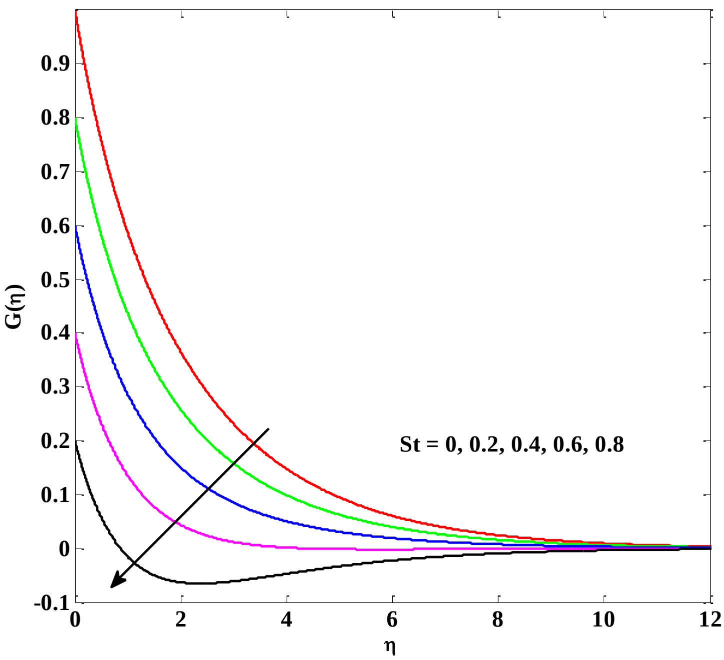

Figure 11(b). Variation of temperature G(η) with η for several values of stratification parameter St in the absence of suction and stagnation.

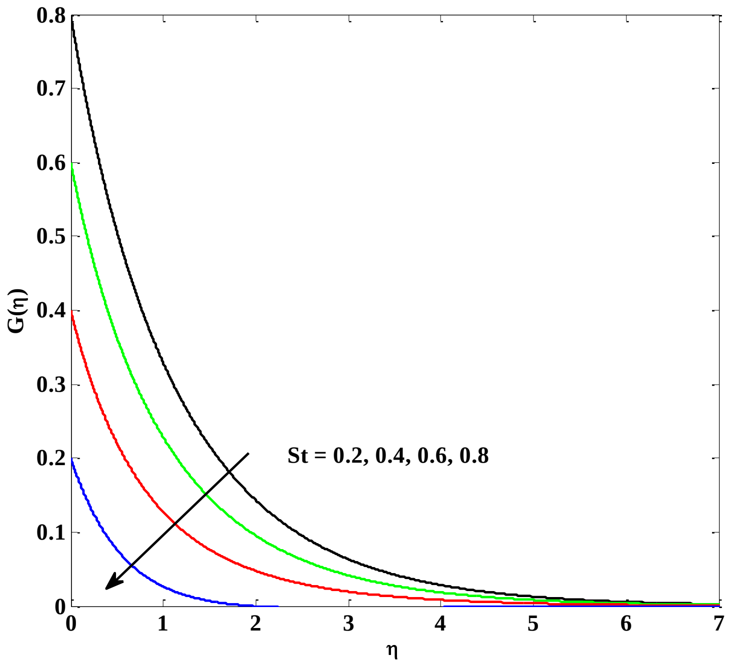

Figure 11(c). Variation of temperature G(η) with η for several values of stratification parameter St in the presence of suction without stagnation.

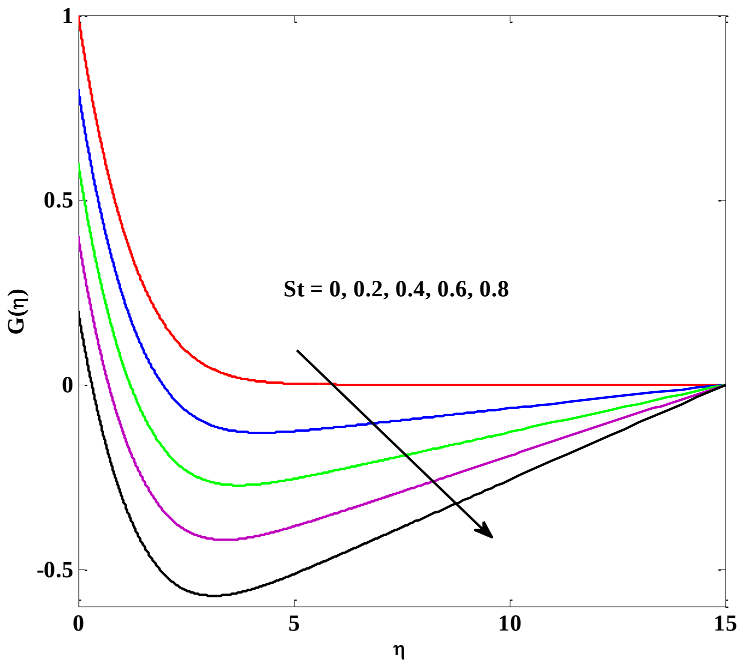

Figure 11(d). Variation of temperature G(η) with η for several values of stratification parameter St in the presence of stagnation without suction.

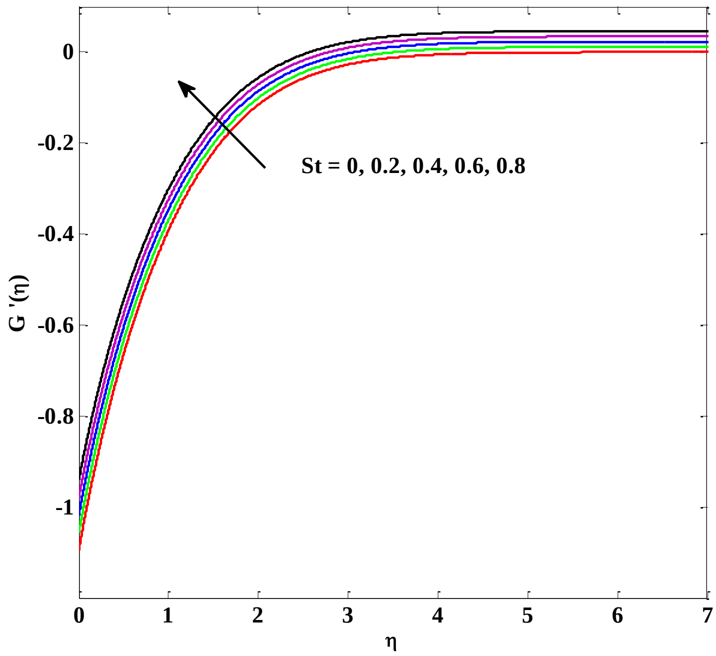

Figure 12(a). Variation of temperature gradient G′(η) with η for several values of stratification parameter St in the presence of suction and stagnation.

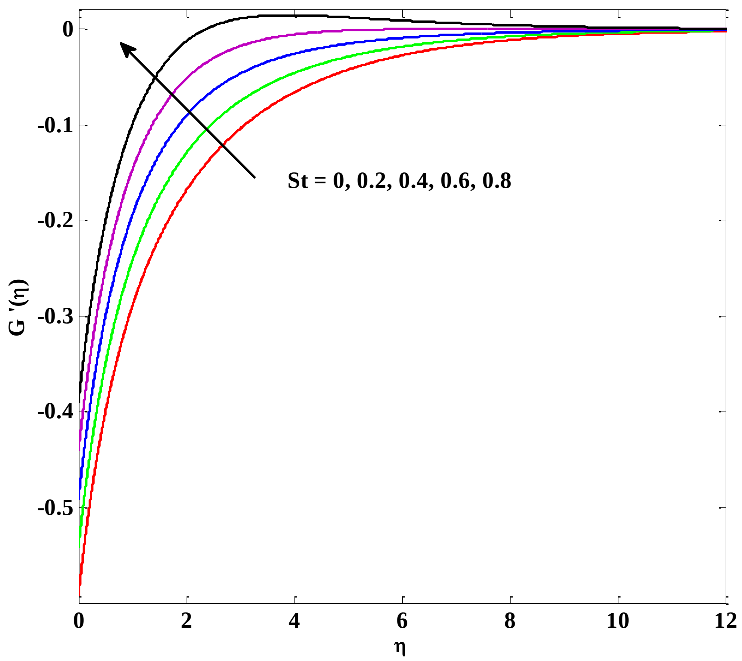

Figure 12(b). Variation of temperature gradient G′(η) with η for several values of stratification parameter St in the absence of suction and stagnation.

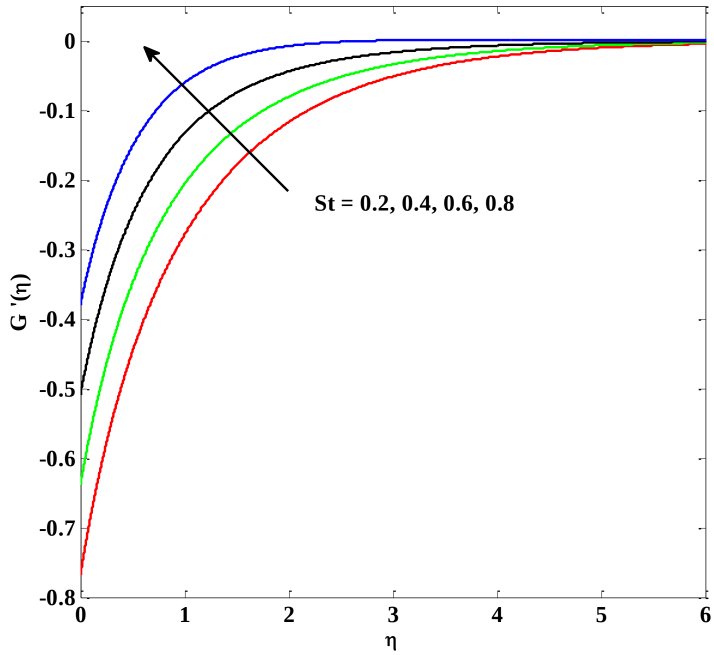

Figure 12(c). Variation of temperature gradient G′(η) with η for several values of stratification parameter St in the presence of suction without stagnation.

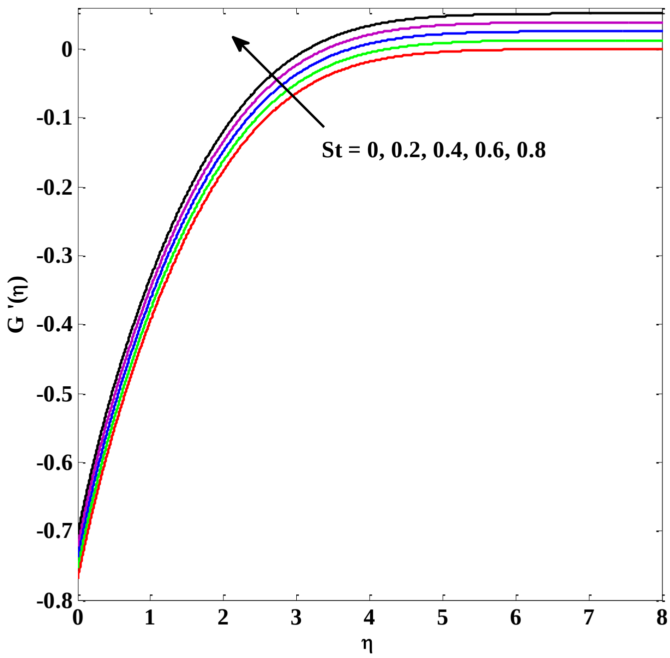

Figure 12(d). Variation of temperature gradient G′(η) with η for several values of stratification parameter St in the presence of stagnation without suction.

The effect of Prandtl number Pr on the heat transfer process is shown by the Fig. 13(a) This figure reveals that as an increase in Prandtl number Pr, the temperature field decreases. An increase in the values of Pr reduces the thermal diffusivity, because Prandtl number is a dimensionless number which is defined as the ratio of momentum diffusivity to thermal diffusivity, that is $Pr = \upsilon/\alpha$. Increasing the values of Pr implies that momentum diffusivity is higher than thermal diffusivity. Therefore thermal boundary layer thickness is a decreasing function of Pr. In general the Prandtl number is used in heat transfer problems to reduce the relative thickening of the momentum and the thermal boundary layers. Also effect of Prandtl number Pr on temperature gradient is shown in Fig. 13(b).

The impact of the thermal radiation parameter R on the temperature profiles is presented in Fig. 14(a). When we increase the value of thermal radiation, it provides more heat to the fluid it causes enhancement in the temperature. Due to the reduction of rate of heat transfer at the surface the thermal boundary layer thickness increases as the value of thermal radiation increases. Also effect of thermal radiation R on temperature gradient is shown in Fig. 14(b).

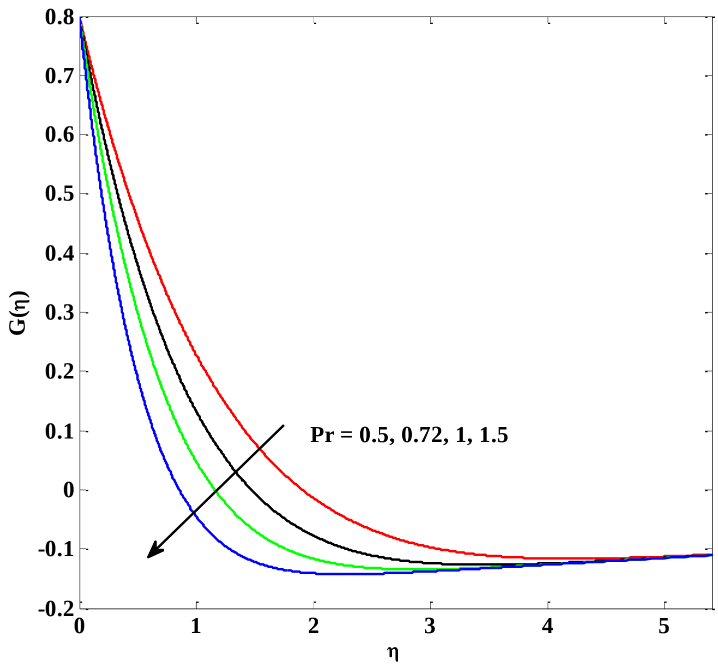

Figure 13(a). Variation of temperature G(η) with η for several values of Prandtl number Pr.

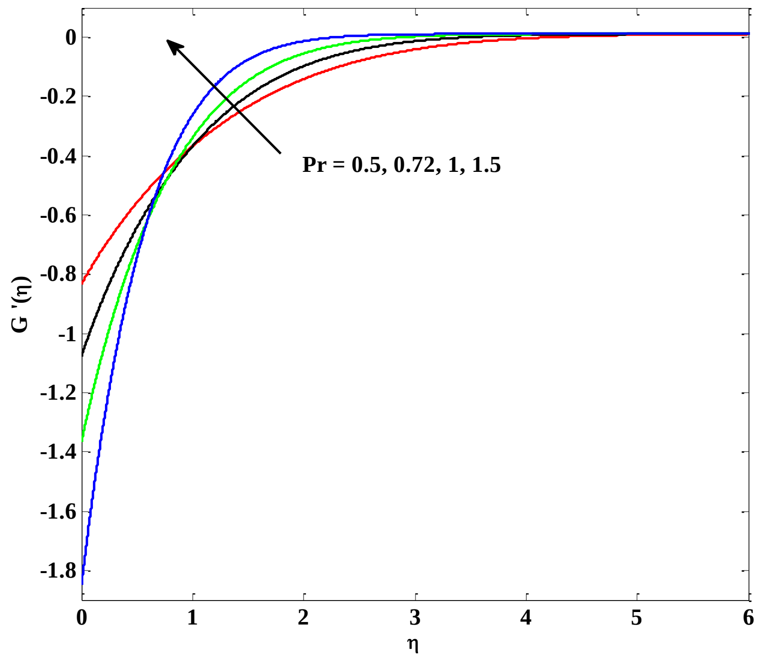

Figure 13(b). Variation of temperature gradient G'(η) with η for several values of Prandtl number Pr.

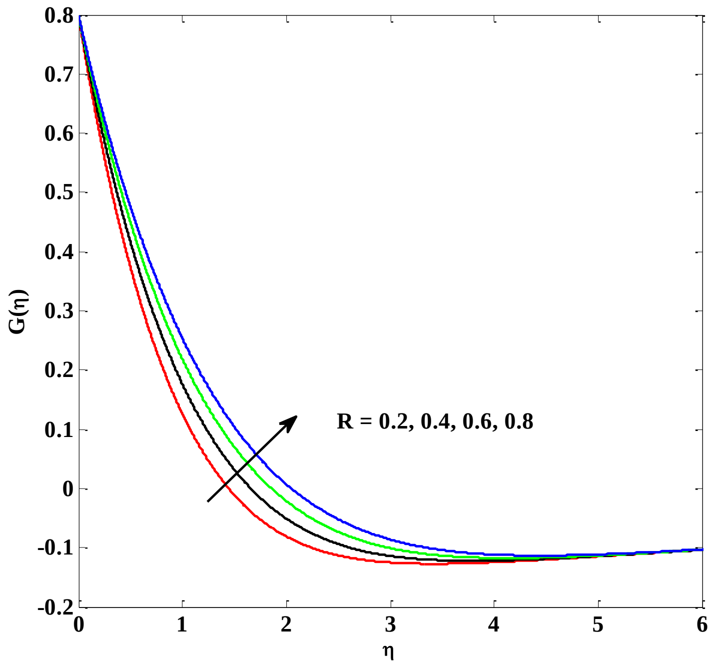

Figure 14(a). Variation of temperature G(η) with η for several values of Radiation parameter R.

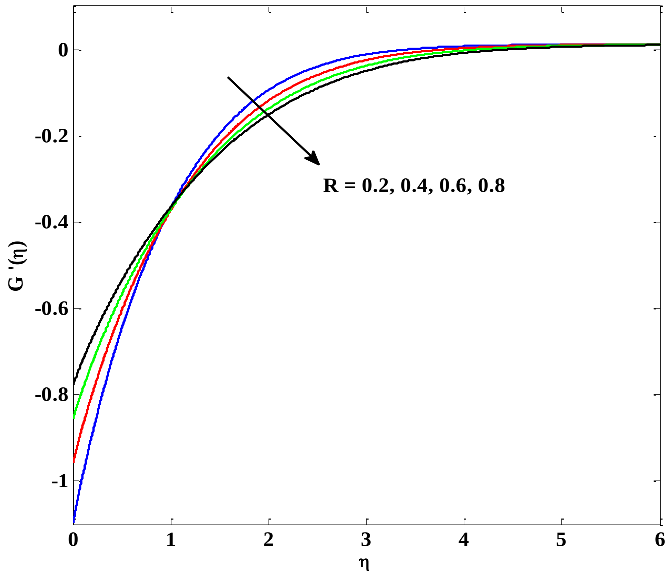

Figure 14(b). Variation of temperature G'(η) with η for several values of Radiation parameter R.

Table 2. Computed values of Skin friction coefficient $\sqrt{2\mathrm{Re}}C_f = \left(f''(0) + \frac{\lambda}{2} f''^2(0) \right)$ for various values of ε, λ, M, S.

ε	λ	M	S	-f ″ (0)
0	0.2	0.1	1	2.3436
0.5				1.3305
1				0.0034
1.5				1.3813
	0			1.1901
	0.1			1.2505
	0.2			1.3305
	0.3			1.4448
		0		1.3158
		0.1		1.3305
		0.2		1.3455

	0.3		1.3606
		0	0.9570
		0.1	0.9886
		0.2	1.0554
		0.3	1.2043

Table 3. Computed values of Local Nussult number $Nu_x\sqrt{\frac{2}{\text{Re}}}(1-St)=-G'(0)$ for various values of Pr, R, St.

Pr	R	St	-G′(0)
1			1.3313
2			2.2495
3			3.0982
	0.2		1.0652
	0.4		0.9289
	0.6		0.8292
		0.2	1.0266
		0.4	0.9599
		0.6	0.8931

VI. CONCLUSIONS

In the present study, we have investigated the MHD stagnation point flow and heat transfer of Williamson fluid over exponential stretching sheet embedded in a thermally stratified medium by employing a finite difference technique known as Keller-Box method. The important findings are concluded as follows.

- With an increase in the Williamson fluid parameter λ the velocity of the fluid decreased, whereas the Skin friction coefficient increased.
- The velocity of the fluid and the boundary layer thickness increases for $\varepsilon < 1$ and velocity increases and the boundary layer thickness decreases for $\varepsilon > 1$ with an increase in ε.

- Thermal boundary layer thickness decreases with an increase in the velocity ratio parameter ε.
- The effect of suction as well as magnetic parameter on the Williamson fluid is to suppress the velocity field which in turn causes the enhancement of the skin-friction coefficient.
- When suction increases the temperature decreases, but the wall temperature gradient increases.
- The thermal boundary layer thickness decreases with the effect of Prandtl number, but the opposite effect is observed with the radiation parameter.
- The temperature decreases with increasing values of the stratification parameter but, the temperature gradient increases.

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